Y. Üçtug Solved Problems 08/04/2015

Q2) a) Rotor Winding Diagram

12 slots, 2-pole, single-layer full-pitched

1	2	3	4	5	6	7	8	9	10	11	12
a_1	a_2	$-c_1$	$-c_2$	b_1	b_2	$-a_1$	$-a_2$	c_1	c_2	$-b_1$	$-b_2$

Stator Winding Diagram

18 slots, 2-pole, double-layer, 10/9 full-pitched

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A_1	A_2	A_3	$-C_1$	$-C_2$	$-C_3$	B_1	B_2	B_3	$-A_4$	$-A_5$	$-A_6$	C_4	C_5	C_6	$-B_4$	$-B_5$	$-B_6$
$-B_3$	A_4	A_5	A_6	$-C_4$	$-C_5$	$-C_6$	B_4	B_5	B_6	$-A_1$	$-A_2$	$-A_3$	C_1	C_2	C_3	$-B_1$	$-B_2$

b) Draw the air-gap mmf distribution produced by the rotor at $t = 1/150$ s.

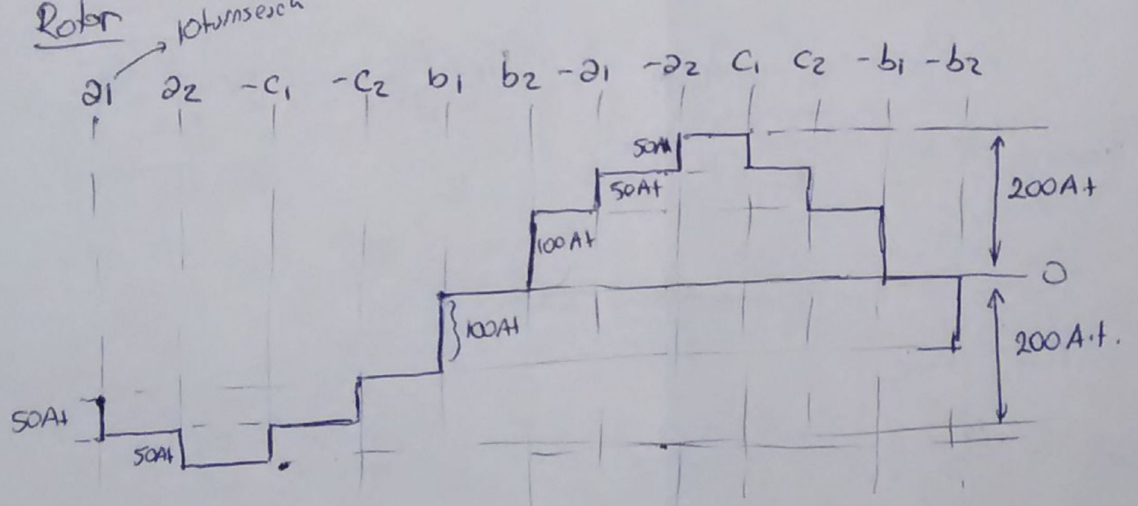
$t = 1/150$

$$I_a(t) = 10\cos(100\pi t) \Rightarrow 10\cos(2\pi/3) = -5A$$

$$I_b(t) = 10\cos\left(100\pi t - \frac{2\pi}{3}\right) \Rightarrow 10A$$

$$I_c(t) = 10\cos\left(100\pi t - \frac{4\pi}{3}\right) \Rightarrow -5A$$

Rotor



c) Calculate the first-harmonic, open-circuit RMS stator voltage when

i) Rotor speed is zero.

ii) Rotor speed is 3000 rpm

iii) " " " -3000 rpm

c) i) First calculate the MMF (peak value of the fundamental $= \hat{F}_1$)

In order to do that, we need to calculate the winding factor (k_w) of the rotor.

$k_w = k_d\, k_c \qquad k_c = 1$ (because it's full pitched)

$k_{wr} \Rightarrow k_{dr} \Rightarrow$ 12 slots $\Rightarrow \alpha = 30°$ ($\frac{360}{12}$, angle between slots) $\quad q = 2$ (slots per pole per phase)

$$k_d = \frac{\sin(q\frac{\alpha}{2})}{q\sin(\frac{\alpha}{2})} = \frac{\sin(2.\frac{30}{2})}{2.\sin(\frac{30}{2})} = 0{,}966$$

$k_{wr} = 0{,}966$

$$\hat{F}_1 = \frac{3}{2} \cdot \frac{4}{\pi} \cdot k_{wr} \frac{20 \cdot 10}{2} = 184{,}5 \text{ At}$$

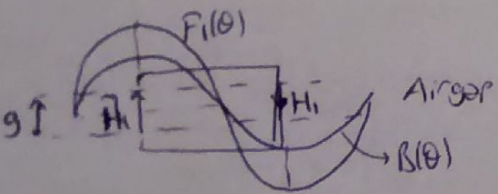
(pole number) I peak

N_{coil} (phase) (2x10)

winding factor

(from 3-phase) (fundamental magnitude of a square wave)

– Then, calculate the flux density ($\hat{B}_1$)

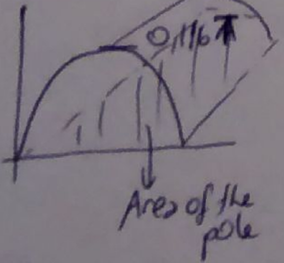


$$\hat{F}_1 = \hat{H}_1 \cdot g \qquad \hat{H}_1 = \frac{\hat{B}_1}{\mu_0}$$

$$\hat{B}_1 = \frac{\mu_0}{g} \cdot \hat{F}_1$$

$$\hat{B}_1 = \frac{4\pi . 10^{-7}}{0{,}002} \cdot 184{,}5 = 0{,}116 \text{ T}$$

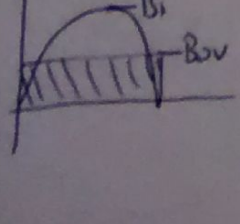
– Then, calculate the flux per-pole.



$\phi = \int B\, dA$

↳ easy way, get the average, multiply by pole area.

$$B_{av} = \frac{2}{\pi} \hat{B}_1$$



$\phi_{pole} = B_{av} . A_{pole}$

$$= \frac{2}{\pi} . \hat{B}_1 . \frac{\overbrace{2\pi . r . l}^{\text{rotor area of cylinder}}}{\underset{\text{pole}}{2}} = \frac{2}{\cancel{\pi}} . 0{,}116 . \frac{2\cancel{\pi} . 0{,}1 . 0{,}3}{\cancel{2}}$$

$$\boxed{\phi_{pole} = 6{,}96 \text{ mWb}}$$

c) Then, in order to calculate the induced stator voltage, we need stator winding factor.

Stator 10/9 pitched $\Rightarrow 180 \cdot \frac{10}{9} = 200°$ coil pitch

$$k_c = \sin\left(\frac{\alpha}{2}\right) = \sin\left(\frac{200}{2}\right) = 0.98481$$

$$k_d = \frac{\sin\left(q\frac{\alpha}{2}\right)}{q\sin\left(\frac{\alpha}{2}\right)} = \frac{\sin\left(3 \cdot \frac{20}{2}\right)}{3 \cdot \sin\left(\frac{20}{2}\right)} = 0.9598$$

$k_w = k_c k_d$

$k_w = 0,9452$

$q=3$ (distributed over 3 slots)

$\alpha = 20$ (18 slots)

$$U_{phase\,(rms)} = \underbrace{4.44}_{\frac{2\pi}{\sqrt{2}}} \cdot \underset{\text{number of turns per phase}}{N_{ph}} \cdot k_w \cdot f \cdot \phi_{pole}$$

$$= 4.44 \cdot \underset{\substack{(15\times6) \\ \text{turns per coils} \quad \text{slots per phase}}}{90} \cdot 0{,}9452 \cdot \underset{(60\pi = \omega)}{50} \cdot 6.96m = \boxed{131{,}4\ V}$$

ii) Rotor currents generate a rotating mmF at 3000 rpm $\left(n = \frac{120 f}{P}\right)$ ← 50Hz

Thus, if the rotor is rotating at 3000 rpm, the frequency of the rotor flux will be 100 Hz, with respect to stator windings. Therefore, the induced voltage will be doubled.

$$U_{phase} = 4.44 \cdot 90 \cdot 0{,}9452 \cdot \underset{(100Hz)}{100} \cdot 6.96mWb$$

$$= 262.8\ V$$

iii) If the rotor is rotating at the opposite direction, the stator winding will just see a constant flux ($f = 0$ Hz), hence no voltage will be induced.

$V_{phase} = 0\ V$ (at -3000 rpm)

3-phase, 50Hz, Y-connected, 380V L-L, negligible stator resistance,

CH-2

Q4

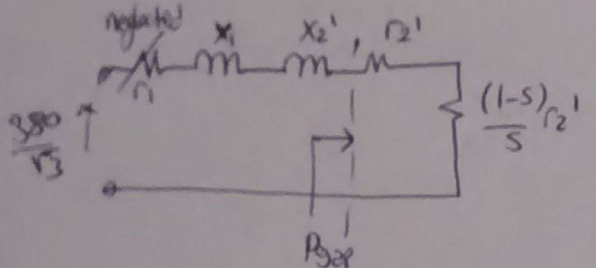
- Motor runs at 745 rpm at no load
- $T_{starting} = 236Nm$
- T_{max} when @ 600rpm

a) Number of poles

$$\frac{120f}{P} \cong 745 \Rightarrow \underline{P = 8 \text{ poles}}$$

b) Referred rotor resistance (r_2')

c) Referred rotor leakage reactance + Stator leakage reactance (X_1+X_2')



At 600rpm, torque is max

$$\hookrightarrow s = \frac{750-600}{750} = 0{,}2$$

Torque is max, when P_{gap} is max. Using max power transfer theorem

$$P_{gap} \max \text{ if } (X_1+X_2') = r_2' + \left(\frac{1-s}{s}\right) r_2'$$

$$(X_1+X_2') = \frac{r_2'}{s} \leftarrow \begin{matrix} s_{max\,T} \\ 0{,}2 \end{matrix}$$

$$\underline{5r_2' = (X_1+X_2')}$$

$T_{start} = 236Nm$ $\quad s=1$

$$T_e = \frac{3V_{th}^2}{\left(\cancel{r_1}+\frac{r_2}{s}\right)^2 + (X_1+X_2)^2} \cdot \frac{r_2}{s} \cdot \frac{1}{w_s} \Rightarrow \frac{3V_{th}^2 \cdot r_2'}{r_2'^2 + (X_1+X_2')^2} \cdot \frac{1}{w_s} = 236$$

$$w_s = \frac{2\pi f_e}{P} \leftarrow \text{pole-pairs}$$

$$w_s = \frac{2\pi \cdot 50}{4}$$

$$\frac{3 \cdot \left(\frac{380}{\sqrt{3}}\right)^2 \cdot r_2'}{26 r_2'^2} \cdot \frac{1}{\frac{2\pi 50}{4}} = 236 \Rightarrow \boxed{r_2' = 0{,}3\Omega}$$

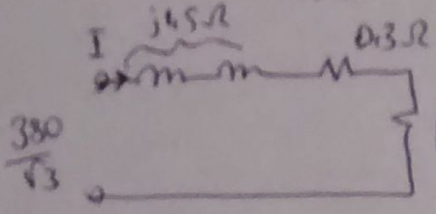
$$\boxed{(X_1+X_2') = 5r_2' = 1{,}5\Omega}$$

CH-2
Q-4

d) $P_{rot} = 1518 W$

Find airgap power, rotor copper loss, internal mechanical power, net output power, efficiency if the rotor is rotating at 712,5 rpm

$$S = \frac{750 - 712{,}5}{750} = 0{,}05$$



$$\left(\frac{1-s}{s}\right) \cdot 0{,}3 = 5{,}7\,\Omega$$

$$I = \frac{380/\sqrt{3}}{\sqrt{6^2 + 1{,}5^2}} = 35{,}47\,A$$

$$P_{gap} = 3 I^2 \cdot \frac{r_2}{s} = 3 \cdot (35{,}47)^2 \cdot 6 = 22{,}65\,kW$$

$$P_{cur} = 3 I^2 \cdot r_2 = 3 \cdot (35{,}47)^2 \cdot 0{,}3 = 1{,}13\,kW$$

$$P_{mech} = P_{gap} - P_{cur} = 21{,}52\,kW$$

$$P_{out} = P_{mech} - P_{loss} = \underline{\underline{20\,kW}}$$

$$\text{Efficiency} = \frac{P_{out}}{P_{in}} = \frac{20\,kW}{22.65\,kW} = \underline{\underline{88{,}3\,\%}}$$